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SOME RELATIONSHIPS BETWEEN MATROIDS AND CONCEPT LATTICES

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Abstract

This paper discusses some relationships between matroids and concept lattices. All the discussion are based on lattice theory.

1 Introduction

Wikipedia on matroid which was accessed on September 5, 2011 [1] pointed out that matroid theory, a branch of combinatorics, dates from the 1930's when van der Waerden in his "Moderns Algebra" first approached linear and algebraic dependence axiomatically and Whitney in his basic paper first used the term matroid.

Additionally, Welsh in [2] indicated that matroid theory borrows extensively from the terminology of linear algebra and graph theory, largely because it is the abstraction of various notions of central importance in these fields. This is not to say that all aspects of combinatorial theory can be covered by the matroid umbrella.

Even though, some researchers discover many new results of matroids applying lattice theory, and besides, some results of matroids are more clearly understood by the use of lattice theory (cf. [1-7]). In addition, using some results of matroids relative to lattice theory, matroid theory are applied in many fields (cf. [1, 2, 8, 9]).

Concept lattice was proposed by Wille [10] in 1982. In addition, Ganter and Wille [11] pointed out that concept lattice theory is a field of applied mathe-

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matics based on the mathematization of concept and conceptual hierarchy, in particular on theory of complete lattices. Thereby, as an effective tool for data analysis and knowledge processing, concept lattice has been applied to various fields (cf. [10-13]).

Though the history and present condition of matroids and concept lattices illustrate that lattice theory is an effective research way on matroids and concept lattices, how to use matroid theory to the study on concept lattice theory has seldom results. If we utilize matroid theory into the study of concept lattices, then we may prescribe that it obtains a new way for concept lattice's research. Approaching to this purpose, the first what we need to do is to find out the relationships between matroids and concept lattices. This paper will do this work. The other works, such as using these relationships to apply some approaches of matroid theory into the discussion of concept lattices, are left rooms to the future.

The paper is organized as follows. Section 2 is to review some preliminaries what we need. In Section 3, using a constructive way, we build up a concept lattice directly from a matroid. Afterwards, we reveal some relationships between matroids and concept lattices by lattice theory.

2 Basic notions and properties

We recall back some basic notions and properties what we need in the sequel. For the others, concept lattices are referred to [11, 12]; lattice theory are seen in [14, 15]; matroids are referred to [2].

2.1 Concept lattice

Definition 2.1.1 [11, p.17] A *context* $\mathbb{K} = (O, P, R)$ consists of two sets O and P and a relation R between O and P. The elements of O are called the *objects* and the elements of P are called the *attributes* of context. In order to express that an object o is in a relation R with an attribute p, we write oRp or $(o, p) \in R$ and read it as "the object o has the attribute p".

[11, p.17] A small context can be represented by a *cross table*, i.e., by a rectangular table the rows of which are headed by the object names and the columns headed by the attribute names. A cross in row g and column m means that the object g has the attribute m.

[11, p.18] For a set $A \subseteq O$ of objects, we define $A' = \{p \in P | xRp$ for all $x \in A\}$. Correspondingly, for a set B of attributes, we define $B' = \{o \in O | oRy$ for all $y \in B\}$.

A *concept* of the context (O, P, R) is a pair (A, B) with $A \subseteq O, B \subseteq P, A' = B$ and $B' = A$. We call A the *extent* and B the *intent* of the concept (A, B).

[11, p.19] If (A_1, B_1) and (A_2, B_2) are concepts of a context (O, P, R), (A_1, B_1) is called a *subconcept* of (A_2, B_2), provided that $A_1 \subseteq A_2$. In this case, (A_2, B_2) is a *superconcept* of (A_1, B_1), and we write $(A_1, B_1) \leq_{\mathbb{K}} (A_2, B_2)$. The relation $\leq_{\mathbb{K}}$ is called the *hierarchical order* (or simply order) of the concepts. The set of all concepts of (O, P, R) ordered in this way is denoted by $\mathcal{B}(O, P, R)$ and is called the *concept lattice* of the context (O, P, R).

Lemma 2.1.1 [11, p.18] If (O, P, R) is a context, $A, A_1, A_2 \subseteq O$ are sets of objects and B, B_1, B_2 are sets of attributes, then

(i) $A_1 \subseteq A_2 \Rightarrow A_2' \subseteq A_1'$; (i)' $B_1 \subseteq B_2 \Rightarrow B_2' \subseteq B_1'$;
(ii) $A \subseteq A''$; (ii)' $B \subseteq B''$;
(iii) $A' = A'''$; (iii)' $B' = B'''$.

Lemma 2.1.2 Let (O, P, R) be a context and $(A_t, B_t) \in \mathcal{B}(O, P, R)$ $(t \in T)$.

(1)[11, p.20] The concept lattice $\mathcal{B}(O, P, R)$ is a complete lattice in which infimum and supremum are given by

$\bigwedge_{t\in T}(A_t, B_t) = (\bigcap_{t\in T} A_t, (\bigcup_{t\in T} B_t)''), \bigvee_{t\in T}(A_t, B_t) = ((\bigcup_{t\in T} A_t)'', \bigcap_{t\in T} B_t)$.

(2)[11, p.20] If V is a complete lattice with $\leq_V$ as its partial order, then $\mathcal{B}(V, V, \leq_V) \cong V$.

Remark 2.1.1 For a context (O, P, R), Lemma 2.1.1 and Definition 2.1.1 show that $(A'', A'), (B', B'') \in \mathcal{B}(O, P, R)$ hold for any $A \subseteq O$ and $B \subseteq P$.

2.2 Lattice theory

Definition 2.2.1 [2, p.45&14, 15] If P is a poset with a unique minimal element, which we will always denote by 0, then an *atom* is an element which covers 0.

[2, p.47&14, 15] A finite lattice L is *semimodular* if for all $x, y \in L$: x and y cover $x \wedge y \Rightarrow x \vee y$ covers x and y.

[2, p.48&14, 15] The *height* of a finite semimodular lattice is $h(I)$, and its elements of height equal to $h(I) - 1$ are called its *coatoms*.

[2, p.51&14, 15] A finite lattice is *geometric* if it is semimodular and every point is the join of atoms.

Remark 2.2.1 (1) By the knowledge in [14, 15], we may describe that for two finite lattices L_1 and L_2, if a map $f : L_1 \to L_2$ is join-isomorphic, then f is a lattice isomorphism.

(2) If two lattices L_1 and L_2 are isomorphic, it is denoted as $L_1 \cong L_2$ in this paper.

(3) Let L be a lattice and $X \subseteq L$. In this paper, sometimes, $\bigvee_{x\in X} x$ and $\bigwedge_{x\in X} x$ will be denoted as $\vee X$ and $\wedge X$ respectively.

2.3 Matroid

Definition 2.3.1 [2, pp.7-8] A *matroid* M is a finite set S and a collection $\mathcal{I}$ of subsets of S (called *independent* sets) such that (I1)-(I3) are satisfied.

(I1) $\emptyset \in \mathcal{I}$.

(I2) If $U \in \mathcal{I}$ and $V \subseteq U$, then $V \in \mathcal{I}$.

(I3) If $U, V \in \mathcal{I}$ with $|U| = |V| + 1$, then there exists $x \in U \setminus V$ such that $V \cup x \in \mathcal{I}$.

Lemma 2.3.1 [2, pp.48-50] If M is a matroid on S, we can associate with M a partially ordered set $L(M)$ whose elements are the closed sets of M ordered by inclusion. Then under the inclusion ordering, $L(M)$ is a geometric lattice.

[2, p.51] A finite lattice L is isomorphic to the lattice of closed sets of a matroid if and only if it is geometric.

[2, p.54] The correspondence between a geometric lattice L and the matroid $M(L)$ on the set of atoms of L is a bijection between the set of finite geometric lattices and the set of simple matroids.

Remark 2.3.1 Let M be a matroid on S with $\mathcal{F}$ as its family of closed sets.

(1) By the closure operator axioms in [2, p.8], M can be represented as $(S, \mathcal{F})$. The definition of closed sets of a matroid is referred to [2, p.8].

(2) There is one and only one simple matroid M_1 on S determined by M (cf. [2, p.54]). The definition of a simple matroid is seen [2, p.13].

Notice 2.3.1 Based on Definition 2.3.1, all the discussion in this paper are finite.

3 Relationships

About the relationships between matroids and concept lattices, it has existed the following results.

(3.1) For a given context (O, P, R), H.Mao in [16] presents some criteria to determine under what conditions, LK or KL is the closure operator of a matroid, where (L, K) is the Galois connection decided by (O, P, R).

(3.2) For a matroid M with $\mathcal{F}$ as the set of closed sets, Remark 2.3.1 points out that M can be simplified as M_1; up to isomorphism, there is one and only one geometric lattice $(\mathcal{F}, \subseteq)$ determined by M_1.

Additionally, Lemma 2.1.2 and [11] describe that up to isomorphism, there is a concept lattice $\mathcal{B}(V, V, \subseteq)$ satisfying $\mathcal{B}(V, V, \subseteq) \cong (\mathcal{F}, \subseteq)$, where $V = (\mathcal{F}, \subseteq)$.

From (3.1) and (3.2), it seems that the relationships between matroids and concept lattices are clear and it has no value to consider to find out the inter-relationships again. However, except the way in (3.2), we may hope to discover another much more direct way to construct a context by a constructive method from a matriod. As a result, once we face a question relative to concept lattices or matroids, we may choose the best from the already made ways to answer the question.

How to find a new way to reveal the relationships between matroids and concept lattices? We may notice that lattice theory is applied in the study on both matroids and concept lattices. Utilizing lattice theory, we have discovered some inter-relationships between matroids and concept lattices such as that in (3.2). Lemma 2.1.2, Lemma 2.3.1 and Remark 2.3.1 illustrate that we may reveal the relationships between geometric lattices and concept lattices when we hope to find out the relationships between matroids and concept lattices. Based on these ideas, in this section, we first provide examples to show that the existence of a context which its concept lattice is geometric (cf. Example 3.1) and conversely, a matroid M can provide a concept lattice $\mathcal{B}(M)$ satisfying $\mathcal{B}(M) \cong (\mathcal{F}(M), \subseteq)$ where $\mathcal{F}(M)$ is the family of closed sets of M (cf. Example 3.2). Afterwards, we will pay our attention to construct a context (O, P, R) directly yielded out of a geometric lattice L. Meanwhile, we prove that the concept lattice $\mathcal{B}(O, P, R)$ is isomorphic to L. After that, we achieve a result expressed the inter-relationships between simple matroids and geometric concept lattices.

Example 3.1 Let (O, P, R) be a context where $O = \{1, 2, 3, 4\}, P = \{a, b, c, d, e\}$ and $R \subseteq O \times P$ shown as Table 3.1. From the table, we may receive: $\{1\}' = \{a, c, e\}$ and $\{a, c, e\}' = \{1\}$; $\{2\}' = \{a, b, e\}$ and $\{a, b, e\}' = \{2\}$; $\{3\}' = \{4\}' = \{b, c, d\}$ and $\{b, c, d\}' = \{3, 4\}$. The diagram of $\mathcal{B}(O, P, R)$ is Figure 3.1.

Table 3.1

	a	b	c	d	e
1	×		×		×
2	×	×			×
3		×	×	×	
4		×	×	×	

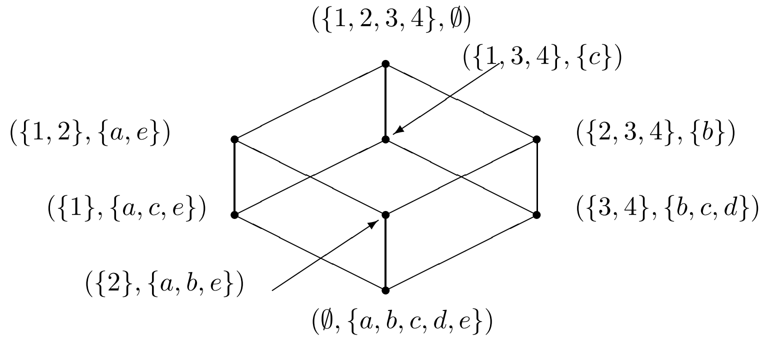

Figure 3.1

Example 3.2 Let $M = (S, \mathcal{F})$ be a matroid with $\mathcal{F}$ as its family of closed sets where $S = \{s_1, s_2, s_3\}$ and $\mathcal{F} = 2^S$. For simplicity, let $a_j = \{s_j\}, (j = 1, 2, 3)$, $c_1 = \{s_1, s_2\}, c_2 = \{s_1, s_3\}, c_3 = \{s_2, s_3\}, 0 = \{\emptyset\}$ and $I = \{S\}$. Then $(\mathcal{F}, \subseteq)$ is shown in Figure 3.2. Evidently, $(\mathcal{F}, \subseteq)$ is geometric. Put $O = \{a_1, a_2, a_3\}$ and $P = \{c_1, c_2, c_3\}$. Define $R \subseteq O \times P$ as $(x, y) \in R \Leftrightarrow x \subseteq y$. Then we receive a context (O, P, R) and the cross table is Table 3.2. From the table, we may achieve $\{a_1\}' = \{c_1, c_2\}, \{a_2\}' = \{c_1, c_3\}, \{a_3\}' = \{c_2, c_3\}$, and $\{c_1, c_2\}' = \{a_1\}, \{c_1, c_3\}' = \{a_2\}, \{c_2, c_3\}' = \{a_3\}$. The diagram of $\mathcal{B}(O, P, R)$ is Figure 3.3.

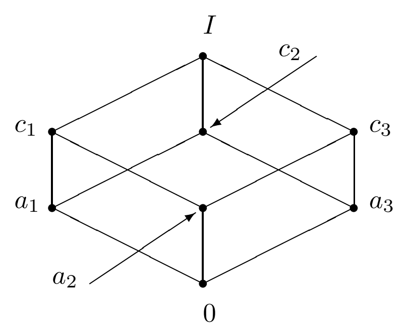

Figure 3.2

Table 3.2

	c_1	c_2	c_3
a_1	×	×	
a_2	×		×
a_3		×	×

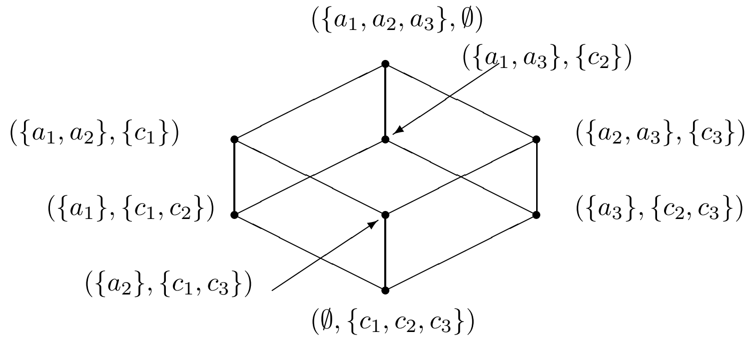

Figure 3.3

For a geometric lattice L with $\leq$ as its partial order, we will prove that there is a context (O, P, R) satisfying $\mathcal{B}(O, P, R) \cong L$.

Let A_L be the set of atoms of L and C_L be the set of coatoms of L. For any $l \in L$, let $A_L(l) = \{a \in A_L | a \leq l\}$ and $C_L(l) = \{c \in C_L | l \leq c\}$.

First, we may sum up some properties of geometric lattices from the definition of a geometric lattice. Let $x, y \in L$. There are the following consequences.

(g1) $x = \vee\{a \in A_L | a \leq x\} = \wedge\{c \in C_L | x \leq c\}$.

(g2) $x = y \Leftrightarrow \{c \in C_L | x \leq c\} = \{c \in C_L | y \leq c\} \Leftrightarrow \{a \in A_L | a \leq x\} = \{a \in A_L | a \leq y\}$.

Second, we establish a context (O, P, R) relative to L.

Put $O = A_L$ and $P = C_L$. Let $R \subseteq O \times P$ be defined as $xRy \Leftrightarrow x \leq y$ in L.

Third, we prove $\mathcal{B}(O, P, R) \cong L$ step by step.

Step 1. To prove: for any $(X, Y) \in \mathcal{B}(O, P, R)$, there is $\{a \in A_L | a \leq \vee X$ in $L\} = X$.

According to $X \subseteq O = A_L$ and the geometric property of L, it infers to $\vee X \in L$ and $x \leq \vee X$ in L for any $x \in X$. That is to say, $\{a \in A_L | a \leq \vee X$ in $L\} \supseteq X$ holds.

Additionally, choose $a \in \{a \in A_L | a \leq \vee X$ in $L\}$.

By $(X, Y) \in \mathcal{B}(O, P, R)$ and Definition 2.1.1, for any $y \in X'$, it obtains $x \leq y$ for all $x \in X$. This follows $\bigvee_{x \in X} x \leq y$. So, $a \leq \vee X \leq y$ holds for any $y \in X'$. Thus, we may arrive at $a \in X'' = X$. This causes $\{a \in A_L | a \leq \vee X$ in $L\} \subseteq X$.

Moreover, it achieves $X = \{a \in A_L | a \leq \vee X$ in $L\}$. Further, $(X, Y) \in \mathcal{B}(O, P, R)$ follows $Y = X' = \{y \in C_L | x \leq y, \forall x \in X\}$.

Step 2. Let $a \in L$. We prove that there is $(X, Y) \in \mathcal{B}(O, P, R)$ satisfying $a = \vee X$.

Put $X = A_L(a)$ and $Y = C_L(a)$. Then, obviously, $X \subseteq O$ and $Y \subseteq P$ are correct. In view of Definition 2.1.1, we receive

$X' = \{y \in P | xRy, \forall x \in X\} = \{y \in P | x \leq y \text{ for all } x \in X\}$.

We may deal with the properties of X' as follows.

Step 2.1. To prove $X' = Y$.

Let $y \in X'$. Then for any $x \in X$, $x \leq y$ holds. This carries out $\vee X \leq y$. By (g1), we may know $a = \vee A_L(a) = \vee X$. Thus, it obtains $a \leq y$. This infers to $y \in C_L(a)$. Furthermore, $X' \subseteq Y$ is correct.

Additionally, let $y_2 \in Y$. Then $a \leq y_2$ holds evidently. By the definition of $A_L(a)$, it gets $x \leq a$ for any $x \in A_L(a)$. Thus, for any $x \in A_L(a) = X$, $x \leq a \leq y_2$ holds, that is, $x \leq y_2$ is true. Hence, we may indicate $y_2 \in X'$.

In one word, $X' = Y$ is true.

Step 2.2. To prove $Y' = X$.

By Definition 2.1.1, we assure

$Y' = \{x \in O | xRy, \forall y \in Y\} = \{x \in A_L | x \leq y, \forall y \in Y\}$.

Let $y_1 \in Y'$. Then, $y_1 \leq y$ holds for any $y \in Y$. Thus, we may obtain $y_1 \leq \wedge Y$. Since L is geometric, by (g1) and (g2), we obtain $\vee X = \vee A_L(a) = a = \wedge C_L(a) = \wedge Y$. Thus, $y_1 \leq a$ is correct. Furthermore, $y_1 \in X$ is true. Therefore, $Y' \subseteq X$ is followed.

Let $x_2 \in X$. Then $x_2 \leq \vee X = \vee A_L(a) = a = \bigwedge_{y \in Y} y$. This follows $x_2 \leq y$ for any $y \in Y$. Thus, $x_2 \in Y'$ is true. Furthermore, $X \subseteq Y'$ holds.

Therefore, $X = Y'$ is followed.

Step 2.3. To prove $X = X''$.

Considered $Y = X'$ by Step 2.1, we may express that $Y' = X''$ holds. Combining with Step 2.2, it follows $X = X''$.

Step 2.4. Combining Step 2.1-Step 2.3 with Remark 2.1.1, we may infer to $(X, Y) = (X'', X') \in \mathcal{B}(O, P, R)$ and $a = \vee X$.

Step 3. Considering Step 1 and Step 2, we may define a map $f : \mathcal{B}(O, P, R) \to L$ as $(X, Y) \mapsto \bigvee_{g \in X} g$. We prove that f is a lattice isomorphic map.

Step 3.1. To prove: f is an injection.

Let $(X_j, Y_j) \in \mathcal{B}(O, P, R)\,(j = 1, 2)$. By Definition 2.1.1, we know $(X_1, Y_1) \neq (X_2, Y_2)$ if and only if $X_1 \neq X_2$. According to $X_j \subseteq O\,(j = 1, 2)$ and Step 1, we may believe that $X_1 \neq X_2$ follows $\bigvee_{g \in X_1} g \neq \bigvee_{g \in X_2} g$ in L. Hence, it yields out that f is an injection.

Step 3.2. To prove: f is surjective.

Let $y_0 \in L$. Utilizing the geometric property of L, it carries out $y_0 = \bigvee_{g \in A_L(y_0)} g$. In view of Step 2, there exists $X = \{g \in A_L | g \leq y_0\}$ satisfying $(X, X') \in \mathcal{B}(O, P, R)$ and $\vee X = y_0$. Further, using the definition of f, we receive $f((X, X')) = \bigvee_{g \in X} g = \bigvee_{g \in A_L(y_0)} g = y_0$.

Therefore, we may state that f is a surjection.

Step 3.3. To prove: f is join-preserved.

Let $(X_j, Y_j) \in \mathcal{B}(O, P, R)\,(j = 1, 2)$. We will prove

$f((X_1, Y_1) \vee (X_2, Y_2)) = f((X_1, Y_1)) \vee f((X_2, Y_2))$.

According to the definition of f, we get $f((X_j, Y_j)) = \bigvee_{g \in X_j} g$, $(j = 1, 2)$. Considered the geometric property of L, we may indicate $(\bigvee_{g \in X_1} g) \vee$

$(\bigvee_{g\in X_2} g) = \bigvee_{g\in X_1\cup X_2} g$. However, the property that f is a surjection and a injection follows that there is one and only one $(X,Y)\in \mathcal{B}(O,P,R)$ satisfying $f((X,Y)) = \bigvee_{g\in X_1\cup X_2} g$, i.e. $f((X,Y)) = \bigvee_{g\in X} g = \bigvee_{g\in X_1\cup X_2} g$. Thus, there is $f((X,Y)) = f((X_1,Y_1)) \vee f((X_2,Y_2))$.

Considered $\bigvee_{g\in X} g = (\bigvee_{g\in X_1} g) \vee (\bigvee_{g\in X_2} g)$, we may carry out $\bigvee_{g\in X_j} g \leq \bigvee_{g\in X_1\cup X_2} g = \bigvee_{g\in X} g, (j = 1,2)$. Taken this result and Step 1 with the geometric property of L, we may point out $X_j \subseteq X, (j = 1,2)$, and so, $X_1\cup X_2 \subseteq X$. Thus, it yields out $(X_1\cup X_2)'' \subseteq X'' = X$ by Lemma 2.1.1. In light of Lemma 2.1.1, we may achieve $X_1\cup X_2 \subseteq (X_1\cup X_2)''$. Hence, there is $\bigvee_{g\in X_1\cup X_2} g \leq \bigvee_{g\in (X_1\cup X_2)''} g \leq \bigvee_{g\in X} g = \bigvee_{g\in X_1\cup X_2} g$ to be correct. That is to say, $\bigvee_{g\in (X_1\cup X_2)''} g = \bigvee_{g\in X} g = f((X,Y))$ holds. According to Remark 2.1.1, it causes $((X_1\cup X_2)'', (X_1\cup X_2)') \in \mathcal{B}(O,P,R)$. Further, by the definition of f, we may believe $f((X_1\cup X_2)'', (X_1\cup X_2)') = \bigvee_{g\in X} g = f((X,Y))$.

In view of Lemma 2.1.2, there is

$$(X_1,Y_1)\vee(X_2,Y_2) = ((X_1\cup X_2)'', Y_1\cap Y_2)) = ((X_1\cup X_2)'', (X_1\cup X_2)').$$

Moreover, we may carry out

$f((X_1,Y_1)\vee(X_2,Y_2)) = f((X_1\cup X_2)'', Y_1\cap Y_2)) = \bigvee_{g\in (X_1\cup X_2)''} g = f((X_1,Y_1))\vee f((X_2,Y_2))$.

Step 3.4. In light of Remark 2.2.1 and Step 3.1-Step 3.3, we may express that f is a lattice isomorphism. That is to say, $\mathcal{B}(O,P,R) \cong L$ is true.

Up to now, for a matroid $M = (E,\mathcal{F})$ with $\mathcal{F}$ as its family of closed sets, using the construction of $(\mathcal{F},\subseteq)$, we provide a method to construct a context (O,P,R) satisfying $\mathcal{B}(O,P,R) \cong (\mathcal{F},\subseteq)$. For simplicity, we denote this concept lattice as $\mathcal{B}(M)$.

Let $\mathcal{B}(O,P,R)$ be a concept lattice and geometric. We will construct a matroid.

Let S be the set of atoms in $\mathcal{B}(O,P,R)$ and $\mathcal{F}_{\mathcal{B}} = \{x|x\in \mathcal{B}(O,P,R)\}$. By the knowledge in [2], $(S,\mathcal{F}_{\mathcal{B}})$ is a simple matroid, in notation $M_{\mathcal{B}}$, satisfying $(\mathcal{F}_{\mathcal{B}},\subseteq) \cong \mathcal{B}(O,P,R)$.

Summing up the above with Section 2.3, we may conclude the following consequence.

Theorem 3.1 Up to isomorphism, the correspondence between a simple matroid M and the concept lattice $\mathcal{B}(M)$ is a bijection between the set of geometric concept lattices and the set of simple matroids.

Proof It is straightforward from $(\mathcal{F},\subseteq) \cong \mathcal{B}(M) \cong (\mathcal{F}_{\mathcal{B}(M)},\subseteq) \cong \mathcal{B}(M(\mathcal{B}(M)))$, where $\mathcal{F}$ is the family of closed sets of M and $\mathcal{F}_{\mathcal{B}(M)}$ is the family of closed sets of the matroid $M(\mathcal{B}(M))$ produced by $\mathcal{B}(M)$.

Constructive way is better when we deal with a question. Hence, we may convince that the way in this paper is better. Evidently, the method here is different from that in [2]. We may express the following views (3.3) and (3.4).

Let L be a geometric lattice with $\leq$ as its partial order, and A_L, C_L as its set of atoms and coatoms respectively.

(3.3) If we use the method in [2] to construct a context relative to L, then $\mathcal{B}(L, L, \leq)$ is the concept lattice and satisfies $\mathcal{B}(L, L, \leq) \cong L$.

We may reveal that to obtain the context $(L, L, \leq)$, it needs to consider all the elements in L with the relation $\leq$. The complexity is $o(n^2)$ where $n = |L|$.

(3.4) If we use the method here, then $\mathcal{B}(A_L, C_L, \leq)$ is the concept lattice and satisfies $\mathcal{B}(A_L, C_L, \leq) \cong L$.

We may explore that to obtain the context $(A_L, C_L, \leq)$, it needs only to consider the relation between A_L and C_L. The complexity is $o(ts)$ where $|A_L| = t$ and $|C_L| = s$. In addition, $A_L \subseteq L$ and $C_L \subseteq L$ will cause that the complexity of building up $(A_L, C_L, \leq)$ is less than that of constructing $(L, L, \leq)$. As a result, obviously, if we adopt the same idea to construct $\mathcal{B}(L, L, \leq)$ and $\mathcal{B}(A_L, C_L, \leq)$ respectively, then the complexity of building up $\mathcal{B}(A_L, C_L, \leq)$ is less than that of constructing $\mathcal{B}(L, L, \leq)$, though there is $\mathcal{B}(L, L, \leq) \cong \mathcal{B}(A_L, C_L, \leq) \cong L$.

Though the method here has some advantages comparing to that in [2] (cf. (3.3) and (3.4) above), we may state that the method in [2] is also a good one because it suits for all lattices not only for geometric lattices. However, if one considers some properties of concept lattices relative to matroids or geometric lattices, we may suggest the researcher to choose the method here because it owns lower complexity so as to achieve his(her) aim easily.

We may hope to discover much more methods to construct contexts from matroids. Applying this way, concept lattice theory is enriched and the applied fields will be generalized.

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