East-West J. of Mathematics: Vol. 10, No 1 (2008) pp. 37-44

Further Results On Weak Semiopenness

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Abstract

In [3] the authors, introduced the notion of weak semiopenness and investigated its fundamental properties. In this paper we investigate some other properties of this type of openness, in this connection, we obtain also a new decomposition of semiopenness

1 Introduction and Preliminaries.

N.Levine [10] and N.Biswas [1], introduced and investigated the notions of semi-open sets and semi-closed sets in topological spaces respectively. Since then, a lot of work has been done using these notions and many interesting results have been obtained (cf, [1], [5], [6], [7], [13], [14]). D.A.Rose [16] and D.A.Rose with D.S.Janckovic [17] have defined the notions of weakly open and weakly closed functions and investigated some of the fundamental properties of these types of functions.

Reciently M.Caldas and G.Navalagi [3] introduced the notion of weak semi-openness as a new generalization of semiopenness. They remarked that weakly semiopen functions are not always semiopen. The present note have as purpose obtain a sufficient condition for a weakly semiopen function to be semi-open, establish relationships between this function and other generalized forms of openness, we obtain a new decomposition of semiopenness and also show that the inverse image surjective of every semicompact set from the codomain are quasi H-closed.

Throughout this paper, (X, τ) and (Y, σ) (or simply, X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly

Key words: Topological space, semi-open sets, α-open sets, weak semi-continuity, weak openness, extremally disconnected.
2000 AMS Mathematics Subject Classification: 54A40, 54C10, 54D10; Secondary: 54C08.

stated. If S is any subset of a space X, then $Cl(S)$ and $Int(S)$ denote the closure and the interior of S respectively. Recall that a set S is called regular open (resp. regular closed) if $S = Int(Cl(S))$ (resp. $S = Cl(Int(S))$. A point $x \in X$ is called a θ-cluster [18] point of S if $S \cap Cl(U) \neq \phi$ for each open set U containing x. The set of all θ-cluster points of S is called the θ-closure of S and is denoted by $Cl_\theta(S)$. Hence , a subset S is called θ-closed [18] if $Cl_\theta(S) = S$. The complement of a θ-closed set is called a θ-open set. The θ-interior of a subset S of X is the union of all open subsets of X whose closures are contained in S, and is denoted by $Int_\theta(S)$. A subset $S \subset X$ is called semi-open [10] (resp. α-open [11]), if $S \subset Cl(Int(S))$ (resp. $S \subset Int(Cl(Int(S))$. The complement of a semi-open set is called a semi-closed [1] set. The family of all semi-open (resp. semi-closed) sets of a space X is denoted by $SO(X, \tau)$ (resp. $SC(X, \tau)$). The intersection of all semi-closed sets containing S is called the semiclosure of S [1, 6] and is denoted by $sCl(S)$. The semiinterior [6] of S is defined by the union of all semi-open sets contained in S and is denoted by $sInt(S)$. A subset S of X is said to be semiregular if it is both semi-open and semi-closed in X.

A space X is called extremally disconnected (E.D) [19] if the closure of each open set in X is open. The space X is called semiconnected [14] if X cannot be expressed as the union of two nonempty disjoint semi-open sets.

A function $f : (X, \tau) \to (Y, \sigma)$ is called:
(i) weakly semicontinuous [9] if for each $x \in X$ and each open subset V of Y containing $f(x)$, there exists a semi-open subset U of X such that $x \in U$ and $f(U) \subset Cl(V)$.
(ii) semiopen [2] (resp. semiclosed [12]) if $f(U) \in SO(Y, \sigma)$ (resp. $f(U) \in SC(Y, \sigma)$) for each open (resp. closed) subset U of X.
(iii) weakly open [16] if $f(U) \subset Int(f(Cl(U)))$ for each open subset U of X.
(iv) weakly closed [17] if $Cl(f(Int(F))) \subset f(F)$ for each closed subset F of X.

2 Weakly semiopen functions

The following characterization was given by M.Caldas and G.Navalagi in [3] as a natural dual to the weak semicontinuity due to A.Kar and P.Bhattacharya [9]

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly semiopen if for each $x \in X$ and each open set U of X containing x, there exists $V \in SO(X, \tau)$ containing $f(x)$ such that $V \subset f(Cl(U))$. We denote a weakly semiopen function by $w.s.o.$ Clearly, every open function is semiopen and every semiopen (resp. weakly open) function is $w.s.o.$ but the converse is not true.

Example 2.1 *Let* $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}, \sigma = \{\emptyset, X, \{c\}\}$ *and* $f : (X, \tau) \to (X, \sigma)$ *be the identity function. Then* f *is w.s.o. but it is not* *weakly open since* $f(\{a\}) \not\subset Int(f(Cl(\{a\})))$.

Example 2.2 *Let* $X = \{a, b\}, \tau = \{\emptyset, X, \{b\}\}, Y = \{x, y\}$ *and* $\sigma = \{\emptyset, Y, \{x\}\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be given by* $f(a) = x$ *and* $f(b) = y$. *Then* f *is clearly w.s.o., but it is not a semiopen function since* $f(b)$ *is not a semi-open set in* Y.

Example 2.3 *(i) Let* $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ *and* $\sigma = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$. *Let* $f : (X, \tau) \to (X, \sigma)$ *be the identity function. Then* f *is weakly open since* $Int(f(Cl(\{a\}))) = \{a, b\}$, $Int(f(Cl(\{c\}))) = \{b, c\}$, *and* $Int(f(Cl(\{a, c\}))) = X$. *To see that* f *is not semiopen, observe that* $U = \{a\}$ *and* $f(U)$ *is not semi-open in* (X, σ) *since* $Cl(Int(f(\{a\}))) = \emptyset$.
(ii) Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$ *and* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Clearly* f *is semiopen. But it is not open.*
(iii) Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ *and* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Clearly* f *is semiopen. But it is not weakly open.*

It is clear, that the following diagram of implications is true

$$\begin{array}{ccccc} & & semiopen\, function & & \\ & \nearrow \not\swarrow & & \not\nwarrow \searrow & \\ open\, function & & \not\uparrow \;\not\downarrow & & w.s.o\, function \\ & \searrow \not\nwarrow & & \not\swarrow \nearrow & \\ & & weakly\, open\, function & & \end{array}$$

Theorem 2.4 *For a function* $f : (X, \tau) \to (Y, \sigma)$, *the following are equivalent [3].*
1) f *is w.s.o.*
2) $f(U) \subset sInt(f(Cl(U)))$ *for every* $U \in \tau$.
3) $f(Int_\theta(A)) \subset sInt(f(A))$ *for every* $A \subset X$.
4) $Int_\theta(f^{-1}(B)) \subset f^{-1}(sInt(B))$ *for every* $B \subset Y$.
5) $f^{-1}(sCl(B)) \subset Cl_\theta(f^{-1}(B))$ *for every* $B \subset Y$
6) $f(Int(F)) \subset sInt(f(F))$ *for every* $F^c \in \tau$.
7) $f(Int(Cl(U))) \subset sInt(f(Cl(U)))$ *for every* $U \in \tau$.
8) $f(U) \subset sInt(f(Cl(U)))$ *for every* $U \in \tau^\alpha$.

Proof $(1) \Rightarrow (2)$: Let U be an open set in X and let $y \in f(U)$. It following from (1) $V \subset f(Cl(U))$ for some V semi-open in Y containing y. Hence we have, $y \in V \subset sInt(f(Cl(U)))$. This shows that $f(U) \subset sInt(f(Cl(U)))$.
$(2) \Rightarrow (1)$: Let $x \in X$ and U be an open set in X with $x \in U$. By (2) $f(x) \in f(U) \subset sInt(f(Cl(U)))$. Let $V = sInt(f(Cl(U)))$. Hence $V \subset f(Cl(U))$, with V containing $f(x)$, i.e., f is a weakly semiopen function.
$(2) \Rightarrow (3)$: Let A be any subset of X and $x \in Int_\theta(A)$. Then , there exists an open set U such that $x \in U \subset Cl(U) \subset A$. Then, $f(x) \in f(U) \subset$

$f(Cl(U)) \subset f(A)$. By (2), $f(U) \subset sInt(f(Cl(U))) \subset sInt(f(A))$. It implies that $f(x) \in sInt(f(A))$. This shows that $x \in f^{-1}(sInt(f(A)))$. Thus $Int_\theta(A) \subset f^{-1}(sInt(f(A)))$, and so, $f(Int_\theta(A)) \subset sInt(f(A))$.
(3) $\Rightarrow$ (2) : Let U be an open set in X. As $U \subset Int_\theta(Cl(U))$ implies ,
$f(U) \subset f(Int_\theta(Cl(U))) \subset sInt(f(Cl(U)))$. Hence $f(U) \subset sInt(f(Cl(U)))$.
(3) $\Rightarrow$ (4) : Let B be any subset of Y. Then by (3), $f(Int_\theta(f^{-1}(B)) \subset sInt(B)$. Therefore $Int_\theta(f^{-1}(B)) \subset f^{-1}(sInt(B))$.
(4) $\Rightarrow$ (3) : This is obvious.
(4) $\Rightarrow$ (5) : Let B be any subset of Y. Using (4), we have $X - Cl_\theta(f^{-1}(B)) = Int_\theta(X - f^{-1}(B)) = Int_\theta(f^{-1}(Y - B)) \subset f^{-1}(sInt(Y - B)) = f^{-1}(Y - sCl(B)) = X-(f^{-1}(sCl(B))$. Therefore, we obtain $f^{-1}(sCl(B)) \subset Cl_\theta(f^{-1}(B))$.
(5) $\Rightarrow$ (4) : Similary we obtain, $X - f^{-1}(sInt(B)) \subset X - Int_\theta(f^{-1}(B))$, for every subset B of Y, i.e., $Int_\theta(f^{-1}(B)) \subset f^{-1}(sInt(B))$.
(2) $\Rightarrow$ (6) $\Rightarrow$ (7) $\Rightarrow$ (8) $\Rightarrow$ (2) : Obvious.

Lemma 2.5 *Jankovic and Reilly [8]*
Let A be a subset of (X, τ). Then the following hold.
1) $sCl(A) = A \cup Int(Cl(A))$.
2) $sInt(A) = A \cap Cl(Int(A))$.

Theorem 2.6 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following are equivalent.*
1) f is w.s.o.
2) $f(U) \subset Cl(Int(f(Cl(U))))$ for every $U \in \tau$.
3) $f(Int_\theta(A)) \subset Cl(Int(f(A)))$ for every $A \subset X$.
4) $Int_\theta(f^{-1}(B)) \subset f^{-1}Cl(Int(B))$ for every $B \subset Y$.
5) $f^{-1}Int(Cl(B)) \subset Cl_\theta(f^{-1}(B))$ for every $B \subset Y$
6) $f(Int(F)) \subset Cl(Int(f(F)))$ for every $F^c \in \tau$.
7) $f(Int(Cl(U))) \subset Cl(Int(f(Cl(U))))$ for every $U \in \tau$.
8) $f(U) \subset Cl(Int(f(Cl(U))))$ for every $U \in \tau^\alpha$.

Proof This follows from Theorem 2.4 and Lemma 2.5.

In 1984 D.A. Rose [16] asked the following question: When a surjection $f : (X, \tau) \to (Y, \sigma)$, is weak openness related to the condition $Cl(f(U)) \subseteq f(Cl(U))$ for each $U \in \tau$?. In [4] an alternative answer was given to this question by proving the following:
If $f : (X, \tau) \to (Y, \sigma)$ is a bijective function, then f is weakly open if and only if $Cl(f(U)) \subseteq f(Cl(U))$ for each $U \in \tau$.

The following theorem is a version of this result for weakly semiopen functions.

Theorem 2.7 *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following statements are equivalent.*
1) f is w.s.o.

2) $sCl(f(U)) \subset f(Cl(U))$ for each $U \in \tau$.
3) $sCl(f(Int(F)) \subset f(F)$ for each F closed of X.

Proof (1) $\Rightarrow$ (3) : Let F be a closed set in X. Then we have $Y - f(F) = f(X - F) \subset sInt(f(Cl(X - F)))$ and so $Y - f(F) \subset Y - sCl(f(Int(F)))$. Hence $f(F) \supset sCl(f(Int(F)))$.
(3) $\Rightarrow$ (2) : Let $U \in \tau$. Since $Cl(U)$ is a closed set and $U \subset Int(Cl(U))$ by (3) we have $sCl(f(U)) \subset sCl(f(Int(Cl(U)))) \subset f(Cl(U))$.
(2) $\Rightarrow$ (3) : Clear.
(3) $\Rightarrow$ (1) : Similar to (1) $\Rightarrow$ (3).

Theorem 2.8 *Let $g : (Y, \sigma) \to (Z, \gamma)$ be an open continuous injective. Then a function $f : (X, \tau) \to (Y, \sigma)$ is w.s.o. if and only if $g\circ f : (X, \tau) \to (Z, \gamma)$ is w.s.o.*

Proof Necessity. Suppose that f is *w.s.o.* Let U be any open set (X, τ). By Theorem 2.6 $f(U) \subset Cl(Int(f(Cl(U))))$. Since g is open and continuous, we have $g(Cl(Int(A)) \subset Cl(Int(g(A)))$ for every subset A of Y. Therefore, we obtain $(g \circ f)(U) \subset Cl(Int(g \circ f)(Cl(U)))$. It follows from Theorem 2.6 that $g \circ f$ is *w.s.o.*
Sufficiency. Suppose that $g \circ f$ is *w.s.o.* Let U be any open set of (X, τ). By Theorem 2.6, $(g \circ f)(U) \subset Cl(Int(g \circ f)(Cl(U)))$. Since g is open and continuous, we have $g^{-1}(Cl(Int(A))) \subset Cl(Int(g^{-1}(A)))$ for every subset A of Z. Moreover, singe g is injective we obtain $f(U) \subset Cl(Int(f(Cl(U))))$. It follows from Theorem 2.6, that f is *w.s.o.*

Theorem 2.9 *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following hold.*
1) If f has the property that for each regular closed set A, $f(A) \in SO(Y, \sigma)$, then f is w.s.o.
2) If f is w.s.o. , then $f(B) \in SO(Y, \sigma)$ for each clopen set B.

Proof (1) Since, for any $U \in \tau$, $Cl(U)$ is a regular closed set. Then $f(U) \subset f(Cl(U)) = sInt(f(Cl(U)))$.
(2) If B is a clopen set, $f(B) \subset sInt(f(B))$, i.e., $f(B)$ is semi-open.

Since if X is an extremally disconnected space, then regular closed sets are precisely the clopen sets, we have the following remark.

Remark 2.10 *If X is an extremally disconnected space, then both converses of Theorem 2.9 are hold since the regular closed sets are precisely the clopen sets.*

Recall that, a space X is said to be hyperconnected [13] if every nonempty open subset of X is dense in X.

Theorem 2.11 *[3] If X is a hyperconnected space, then a function $f : (X, \tau) \to (Y, \sigma)$ is weakly semiopen if and only if $f(X)$ is semi-open in Y.*

Proof The sufficiency is clear. For the necessity observe that for any open subset U of X, $f(U) \subset f(X) = sInt(f(X)) = sInt(f(Cl(U)))$.

Lemma 2.12 *If $f : (X, \tau) \to (Y, \sigma)$ is w.s.o., then for each $U \in \tau$, we have $f(U) \subset f(Cl(U)) \cap V$, where V is a semi-open subset of Y.*

Proof Take $V = sInt(f(Cl(U)))$.

The Lemma 2.12, suggests the following generalization of semiopenness.

Definition 1 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be relatively weakly semiopen provided that $f(U)$ is semi-open in the subspace $f(Cl(U))$ for every open subset U of X.*

If a function $f : (X, \tau) \to (Y, \sigma)$ is semiopen , then for each $U \in \tau$, $f(U) \in SO(Y)$ and since $f(U) \subset f(Cl(U))$, then we can see that $f(U)$ is also a semi-open subset of $f(Cl(U))$. Therefore the following theorem has been established.

Theorem 2.13 *If $f : (X, \tau) \to (Y, \sigma)$ is semiopen, then f is relatively weakly semiopen.*

The significance of relative weakly semiopen is that it yields a decomposition of semiopenness with weakly semiopenness as the other factor.

Theorem 2.14 *A function $f : (X, \tau) \to (Y, \sigma)$ is semiopen if and only if f is w.s.o. and relatively weakly semiopen.*

Proof The necessity is given of Theorem 2.13 and of the fact that every semiopen function is *w.s.o.* We prove the sufficiency. Assume f is *w.s.o.* and relatively weakly semiopen. Let U be an open set in X. Since f is relatively weakly semiopen, we have $f(U) = f(Cl(U)) \cap V$, where V is a semiopen set of Y. Let $y \in f(U)$. By the fact that f is *w.s.o.*, it follows that $f(U) = f(U) \cap V \subset sInt(f(Cl(U)) \cap V = sInt(f(Cl(U))) \cap sInt(V) = sInt(f(Cl(U) \cap V) = sInt(f(U))$. Therefore $f(U)$ is semi-open.

Now, we define an additional near semiopen condition which combined with weak semiopenness imply semiopenness.

Definition 2 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to satisfy the weakly semiopen interiority condition if $sInt(f(Cl(U))) \subset f(U)$ for every open subset U of X.*

Example 2.15 *semiopenness does not imply weakly semiopenness interiority condition.*
Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, [a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function and let $U = \{a\}$. Since $sInt(f(Cl(U))) = sInt(f(X)) = Y \not\subset f(U) = \{a\}$, f does not satisfy the weakly semiopen interiority condition. However, f is clearly semiopen

Theorem 2.16 *If $f : (X,\tau) \to (Y,\sigma)$ is w.s.o. and satisfies the weakly semiopen interiority condition, then f is semiopen.*

Proof Let $f : (X,\tau) \to (Y,\sigma)$ be a *w.s.o.* function satisfying the weakly semiopen interiority condition. Let U be a given open subset of X. By weakly semiopenness we have that $f(U) \subset sInt(f(Cl(U)))$ (Theorem 2.4). By weakly semiopenness interiority condition, we have that $sInt(f(Cl(U))) \subset f(U)$. Hence $sInt(f(Cl(U))) = f(U)$ and therefore $f(U)$ is semi -open in Y.

The Example 2.15, shows that neither of these interiority conditions yields a decomposition of semiopenness.

Lemma 2.17 *Let $f : (X,\tau) \to (Y,\sigma)$ be a bijective w.s.o. function. If U is a clopen subset of (X,τ), then $f(U)$ is semiregular in (Y,σ).*

Proof It follows from Theorem 2.4(2) and Theorem 2.7(2).

Theorem 2.18 *If $f : (X,\tau) \to (Y,\sigma)$ is an injective w.s.o. function of a space (X,τ) onto a semiconnected space (Y,σ), then (X,τ) is connected.*

Proof Let us assume that X is not connected. Then there exist nonempty open sets U_1 and U_2 such that $U_1 \cap U_2 = \emptyset$ and $U_1 \cup U_2 = X$. Therefore U_1 and U_2 are clopen in (X,τ) and by Lemma 2.17 $f(U_i) \in SO(Y,\sigma)$ for $i = 1,2$. Moreover, we have $f(U_1) \cap f(U_2) = \emptyset$ and $f(U_1) \cup f(U_2) = Y$. Since f is bijective , $f(U_i)$ is nonempty for $i = 1,2$. This indicates that (Y,σ) is not semi-connected. This is a contradiction.

Recall that a subset of a topological space is called closure compact (or quasi H-closed [18]) (resp. semicompact [15]) if each open cover of the set (resp. semi-open cover of the set) contain a finite subcollection whose closures cover the set (resp.contain a finite subcollection that cover the set).

Theorem 2.19 *Let $f : (X,\tau) \to (Y,\sigma)$ be surjective w.s.o. and let K be a semicompact set of Y, then $f^{-1}(K)$ is a closure compact subset of X.*

Proof Let $\Lambda = \{V_\beta : \beta \in I\}$, I being the index set be a open cover of $f^{-1}(K)$ and set $T = \{U \in \Lambda : U \cap f^{-1}(K) \neq \emptyset\}$. Then T is an open cover of $f^{-1}(K)$. For each $y \in K$, $f^{-1}(y) \in U_y$ for some $U_y \in T$. By weakly semiopenness of f, there exists a $W_y \in SO(Y,\sigma)$ containing y such that $W_y \subset f(Cl(U_y))$. The collection $\{W_y : y \in K\}$ is a semi-open cover of K and so there is a finite subcover $\{W_y : y \in K_0\}$ where K_0 is a finite subset of K. Clearly $\{Cl(U_y) : y \in K_0\}$ covers $f^{-1}(K)$, or $f^{-1}(K)$ is a closure compact subset in X.

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